AP CALCULUS AB Homework 4.2b Name:
DR. PAUL L. BAILEY Wednesday, December 18, 2019

Write your homework *neatly, in pencil,* on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem,* or at least enough of it so that your work is readable. When appropriate, *write in sentences.*

The main theory of section 4.2 is summarized below.

Theorem 1. (Rolle's Theorem)
Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b). Suppose that $f(a) = f(b) = 0$. Then there exists $c \in (a, b)$ such that $f'(c) = 0$.

Theorem 2. (Mean Value Theorem (MVT))
Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b). Then there exists $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Corollary 1. *Let f be continuous on a closed interval $[a, b]$ and differentiable on (a, b). If $f'(x) = 0$ for all $x \in (a, b)$, then $f(x) = f(a) = f(b)$ for all $x \in [a, b]$.*

Corollary 2. *Let f and g be continuous on a closed interval $[a, b]$ and differentiable on (a, b). Suppose $f'(x) = g'(x)$ for all $x \in (a, b)$. Then there exists $C \in \mathbb{R}$ such that $g(x) = f(x) + C$ for all $x \in [a, b]$.*

Problem 1 (Thomas §4.2 # 23). Suppose that $f(1) = 3$ and that $f'(x) = 0$ for all $x \in (0, 2)$. Must $f(x) = 3$ for all $x \in (0, 2)$? for all $x \in [0, 2]$? Give reasons for your answer.

Problem 2 (Thomas §4.2 # 24). Suppose that $f(0) = 5$ and that $f'(x) = 2$ for all $x \in (-2, 2)$. Must $f(x) = 2x + 5$ for all $x \in (-2, 2)$? Give reasons for your answer.

Problem 3 (Thomas §4.2 # 27). Find all possible functions with the given derivative.

(a) x

(b) x^2

(c) x^3

Problem 4 (Thomas §4.2 # 28). Find all possible functions with the given derivative.

(a) $2x$

(b) $2x - 1$

(c) $3x^2 + 2x - 1$

Problem 5 (Thomas §4.2 # 41). A body moves with acceleration $a = d^2s/dt$, initial velocity $v(0)$, and initial position $s(0)$ along a coordinate line, where

$$a = 32, v_0 = 20, s_0 = 5$$

Find the body's position at time t.

Problem 6. Compute dy/dx. Simplify.

(a) $y = \dfrac{x^2 + 3x - 1}{x - 2}$

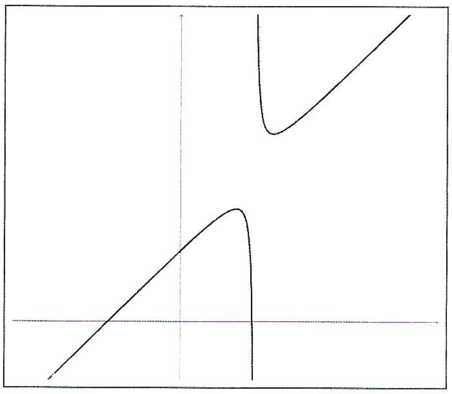
(b) $y = \dfrac{x^2 - 4}{x - 2}$

(c) $y = \sqrt{\sin(x) + x^2}$

(d) $y = \sec^2 x - \tan^2 x$

Problem 7. Let

$$f(x) = \frac{x^2 - 15}{x - 4}.$$



(a) Solve $f'(x) = 0$.

(b) Find the domain and range of f.

(c) The graph of f has to linear asymptotes. Write the equations for these lines.

Problem 8. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \sin(1 + x^2)$. Find all $x \in \mathbb{R}$ such that f is differentiable at x.

Problem 9 (Thomas Ch 2 Practice # 15). Find

$$\lim_{x \to 0} \frac{\frac{1}{2+x} - \frac{1}{x}}{x}.$$

Problem 10 (Thomas §2.6 # 47). Show that the equation

$$x^3 - 15x + 1 = 0$$

has three solutions in the interval $[-4, 4]$. (Hint: use IVT.)

HW 4.2b AP Calculus AB

Key

#1 $f(1)=3$, $f'(x)=0$ all $x\in(0,2)$

Can~~not~~ conclude $f(x)=3$ all $x\in(0,2)$ since it is ~~not~~ given $f(1)=3$ and diff $\Rightarrow$ cont on $(0,2)$.
But cannot conclude $f(0)=3$, don't know continuity.

#2 Yes: By Cor 2, $f'(x)=2 \Rightarrow f(x)=2x+C$.
since $f(0)=5$, $f(x)=2x+5$

#3
ⓐ $\frac{x^2}{2}+C=f \Leftarrow f'=x$
ⓑ $\frac{x^3}{3}+C=f \Leftarrow f'=x^2$
ⓒ $\frac{x^4}{4}+C=f \Leftarrow f'=x^3$

#4
ⓐ $f'=2x \Rightarrow f=x^2+C$
ⓑ $f'=2x-1 \Rightarrow f=x^2-x+C$
ⓒ $f'=3x^2+2x-1 \Rightarrow f=x^3+x^2-x+C$

#5 $a=v'=s''$ $\quad a=32,\ v_0=20,\ s_0=5$
$v'=32 \Rightarrow v=32t+v_0=32t+20$
$s'=32t+20 \Rightarrow s=16t^2+20t+5$

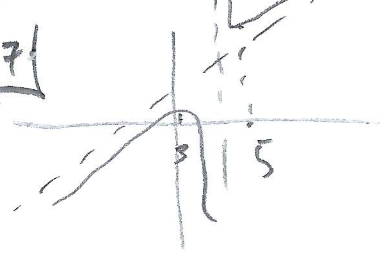
#6
ⓐ $y=\frac{x^2+3x-1}{x-2}$ $\quad y'=\frac{(2x+3)(x-2)-(x^2+3x-1)}{(x-2)^2}=\frac{2x^2-x-6-x^2-3x+1}{(x-2)^2}$
$=\frac{x^2-4x-5}{(x-2)^2}$

ⓑ $y=\frac{x^2-4}{x-2}=x+2 \Rightarrow y'=1$

ⓒ $y=\sqrt{\sin x+x^2}$ $\quad y'=\frac{1}{2\sqrt{\sin x+x^2}}(\cos x+2x)$

ⓓ $y=\sec^2 x-\tan^2 x=1 \Rightarrow y'=0$

#7



$$y = \frac{x^2-15}{x-4}$$

$$y' = \frac{2x(x-4) - (x^2-15)}{(x-4)^2} = \frac{2x^2-8x-x^2+15}{(x-4)^2} = \frac{x^2-8x+15}{(x-4)^2}$$

ⓐ $y'=0 \Rightarrow x=3$ or $x=5$

ⓑ $\text{dom} f = \mathbb{R} \setminus \{4\}$. range $f = (-\infty, 6] \cup [10, \infty)$

$f(3) = \frac{-6}{-1} = 6$

$f(5) = 10$

ⓒ $\frac{x^2-15}{x-4} = \frac{x^2-16+1}{x-4} = (x+4) + \frac{1}{x-4}$

Asymptotes: $x=4$, $y=x+4$

#8 $f(x) = \sin(1+x^2)$ is diff on $\mathbb{R}$

#9 $\lim\limits_{x\to 0} \frac{\frac{1}{2+x} - \frac{1}{x}}{x} = \lim \frac{x-(2+x)}{x^2(2+x)} = \lim \frac{-2}{x(x+2)}$ DNE.

#10 Let $f(x) = x^3 - 15x + 1$

$f(0) = 1$ $\quad f(1) = -13$

$f(-1) = 15$ $\quad f(2) < 0$

$f(-2) = -8+30+1>0$ $\quad f(3) = 27-45+1<0$

$f(-3) = -27+45+1>0$ $\quad f(4) = 64-60+1>0$

$f(-4) = -64+60+1<0$

So, f has a zero in the intervals $(-4,-3)$, $(0,1)$, $(3,4)$